

# Stable Langmuir solitons in plasma with diatomic ions

Maxim Dvornikov<sup>a,b</sup>

<sup>a</sup>*Institute of Physics, University of São Paulo,  
CP 66318, CEP 05315-970 São Paulo, SP, Brazil*

<sup>b</sup>*Pushkov Institute of Terrestrial Magnetism, Ionosphere  
and Radiowave Propagation (IZMIRAN),  
142190 Troitsk, Moscow Region, Russia*

---

## Abstract

We study stable axially and spherically symmetric spatial solitons in plasma with diatomic ions. The stability of a soliton against the collapse is provided by the interaction of induced electric dipole moments of ions with rapidly oscillating electric field of a plasmoid. We derive the new cubic-quintic nonlinear Schrödinger equation which governs the soliton dynamics and numerically solve it. The stability of solitons is analyzed using the Vakhitov-Kolokolov criterion. Then we discuss the possibility of implementation of such plasmoids in a realistic atmospheric plasma. In particular, we suggest that axially and spherically symmetric Langmuir solitons is a theoretical model of long-lived atmospheric plasma structures.

*Keywords:* spatial Langmuir soliton, induced electric dipole moment, long-lived atmospheric plasma structure

---

## 1. Introduction

Stable spatial solitons are observed in the studies of optical phenomena [1], in solid states physics [2], and in the plasma research [3]. Typically the stability of a soliton is provided by a certain nonlinearity. For example, spatial plasma solitons, described in frames of the classical electrodynamics, can exist due to the combined action of electron-ion and electron-electron nonlinear interactions. The former one was found in Ref. [4] to be focusing, whereas the latter interaction can be defocusing [5, 6]. Recently it was

---

*Email address:* maxim.dvornikov@usp.br (Maxim Dvornikov)

suggested [7] that, if one accounts for the additional quantum pressure of electron gas, it may explain the appearance of spatial Langmuir solitons in dense plasmas.

In the present work we shall study the existence of stable Langmuir solitons in a plasma with ions possessing induced electric dipole moments (EDM). We shall demonstrate that the interaction of ion's EDM with a rapidly varying electric field of a plasma oscillation results in the arrest of the Langmuir wave collapse. Thus the existence of a spatial soliton becomes possible.

In our analysis we shall choose a plasma with diatomic ions, which corresponds to a realistic atmospheric plasma. Thus our results may be applied for the theoretical description of long-lived plasma structures observed in the atmosphere [8]. It is interesting to notice that the role of EDM of charged particles for the explanation of the stability of atmospheric plasmoids was also discussed previously in Ref. [9].

This work is organized as follows. In Sec. 2 we consider the general description of nonlinear waves in plasma. We introduce the new ponderomotive force, associated with EDM, which acts on the ion component of plasma. Then we derive a system of nonlinear equations for the electric field amplitude and the perturbation of the ion density. In Sec. 3 we reduce this system to a single nonlinear Schrödinger equation (NLSE) for the envelope of the electric field containing cubic and quintic terms. This equation is analyzed numerically for the case of a radial plasma oscillation. We show that both axially and spherically symmetric stable solitons are possible. Then, in Sec. 4, we discuss a possible application of our model of stable spatial solitons to the description of long-lived atmospheric plasmoids. Finally, in Sec. 5, we briefly summarize our results. The calculation of the permittivity of a gas with diatomic nonpolar molecules in an external electric field is presented in Appendix A.

## 2. Nonlinear Langmuir oscillations in plasma

In this section we shall derive the basic nonlinear equations for the description of Langmuir waves in plasma accounting for a ponderomotive force acting on nonpolar diatomic ions.

If we study electrostatic plasma oscillations, i.e. when the magnetic field is zero, the motion of electron component of plasma obeys the following

hydrodynamics equations:

$$\begin{aligned}\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) &= 0, \\ \frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \nabla) \mathbf{v}_e &= -\frac{e}{m} \mathbf{E} - \frac{1}{n_e m} \nabla p,\end{aligned}\tag{1}$$

where  $n_e$  is the number density of electrons,  $\mathbf{v}_e$  is the electron velocity,  $m$  is the mass of an electron,  $e > 0$  is the proton charge,  $\mathbf{E}$  is the strength of the electric field, and  $p$  is the pressure. We should add to Eq. (1) the corresponding Maxwell and the Poisson equations for the electric field evolution,

$$\begin{aligned}\frac{\partial \mathbf{E}}{\partial t} &= 4\pi e(n_e \mathbf{v}_e - n_i \mathbf{v}_i), \\ (\nabla \cdot \mathbf{E}) &= -4\pi e(n_e - n_i),\end{aligned}\tag{2}$$

where  $n_i$  is the ion number density and  $\mathbf{v}_i$  is the ion velocity.

In the zeroth approximation only electrons participate in a plasma oscillation, with the number density of ions being approximately constant  $n_i \approx n_0 = \text{const}$ . Thus we may present the electric field in the form,

$$\mathbf{E} = \mathbf{E}_1 e^{-i\omega_e t} + \mathbf{E}_1^* e^{i\omega_e t} + \dots,\tag{3}$$

where  $\omega_e = \sqrt{4\pi e^2 n_0 / m}$  is the Langmuir frequency for electrons and  $\mathbf{E}_1$  is the amplitude of an oscillation.

In a realistic situation ions will also participate in a plasma oscillation. Thus the ion density becomes  $n_i = n_0 + n(\mathbf{r}, t)$ , where  $|n| \ll n_0$ . It leads to the appearance of higher harmonics omitted in Eq. (3). The hydrodynamic equations for the description of the ions evolution has the form [6],

$$\begin{aligned}\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) &= 0, \\ \frac{\partial \mathbf{v}_i}{\partial t} &= -\frac{e}{M} \nabla \varphi_{\text{eff}} + \frac{1}{n_i M} \mathbf{f}_{\text{pol}},\end{aligned}\tag{4}$$

where  $M$  is the ion mass,  $\varphi_{\text{eff}}$  is the effective potential which obeys the following Poisson equation [6]:

$$\nabla^2 \varphi_{\text{eff}} = 4\pi e \left( \frac{\nabla^2 U_{\text{pm}}}{4\pi e^2} + r_D^2 \nabla^2 n \right).\tag{5}$$

Here  $r_D = \sqrt{T_e/4\pi e^2 n_0}$  is the Debye length,  $T_e$  is the electron temperature,  $U_{\text{pm}} = |\mathbf{E}_1|^2/4\pi$  is the potential of the ponderomotive force which acts on a charged particle in a rapidly oscillating electric field (3).

In Eq. (4) we include the volume density of the external force  $\mathbf{f}_{\text{pol}}$  which acts on the ion component of plasma. Let us suggest that ions are non-polar molecules which can acquire EDM  $p_i = \alpha_{ij} E_j$  in an external electric field. Here  $(\alpha_{ij})$  is the polarizability tensor of an ion. If an ion is diatomic and possesses an axial symmetry, one can always reduce this tensor to the diagonal form,  $(\alpha_{ij}) = \text{diag}(\alpha_\perp, \alpha_\perp, \alpha_\parallel)$ , where  $\alpha_\perp$  and  $\alpha_\parallel$  are transversal and longitudinal polarizabilities. Using the results of Ref. [10] and Eq. (A.5), we get the ponderomotive force  $\mathbf{f}_{\text{pol}}$  as

$$\mathbf{f}_{\text{pol}} = \frac{1}{8\pi} \left[ \nabla \left( n_i \frac{\partial \varepsilon}{\partial n_i} \mathbf{E}^2 \right) - \mathbf{E}^2 \nabla \varepsilon \right] = n_i \left[ \langle \alpha \rangle + \frac{4}{45} \frac{(\Delta \alpha \mathbf{E})^2}{T_i} \right] \nabla \mathbf{E}^2, \quad (6)$$

where  $\varepsilon$  is the permittivity of the ion component of plasma,  $T_i$  is the ion temperature,  $\langle \alpha \rangle = (2\alpha_\perp + \alpha_\parallel)/3$  is the mean polarizability of an ion, and  $\Delta \alpha = \alpha_\parallel - \alpha_\perp$ .

Combining Eqs. (1)-(6) we get the following nonlinear coupled equations for the electric field,

$$i\dot{\mathbf{E}} + \frac{3}{2}\omega_e r_D \nabla(\nabla \mathbf{E}) - \frac{\omega_e}{2n_0} n \mathbf{E} = \mathbf{0}, \quad (7)$$

and for the perturbation of the ion density,

$$\left( \frac{\partial^2}{\partial t^2} - c_s^2 \nabla^2 \right) n = \frac{1}{4\pi M} \nabla^2 |\mathbf{E}|^2 - \frac{4}{15} \frac{(\Delta \alpha)^2 n_0}{M T_i} \nabla^2 |\mathbf{E}|^4, \quad (8)$$

where  $c_s = \sqrt{T_e/M}$  is the sound velocity in plasma. Note that in Eqs. (7) and (8) we omit the index “1” in the amplitude of the electric field, i.e.  $\mathbf{E}_1 \equiv \mathbf{E}$ , in order not to encumber the formulas.

It should be noticed that in Eq. (8) we neglect the contribution of the ion temperature to the sound velocity since typically  $T_i \ll T_e$ . However we keep the ion temperature in the nonlinear term  $\sim \nabla^2 |\mathbf{E}|^4$  since, as we will see in Sec. 3, it is this term which is responsible for the soliton stabilization. In the rhs of Eq. (8) we also neglect term  $\sim -n_0 \langle \alpha \rangle \nabla^2 |\mathbf{E}|^2/M$ , which is small compared to the contribution of the Miller force. Moreover Eq. (8) we take into account that  $\langle \mathbf{E}^4 \rangle = 6|\mathbf{E}_1|^4$  while averaging over the time interval  $\sim 1/\omega_e$ .

### 3. Cubic-quintic nonlinear Schrödinger equation

In this section we shall derive the nonlinear Schrödinger equation for the amplitude of the electric field. This equation will be considered in a particular case of a radial plasma oscillation. We shall numerically analyze the characteristics of the corresponding Langmuir solitons. In particular, their stability will be examined.

Let us suggest that the density variation in Eq. (8) is slow, i.e.  $\partial^2 n / \partial t^2 \ll c_s^2 \nabla^2 n$ . In this subsonic regime Eqs. (7) and (8) can be cast in a single NLSE,

$$i\dot{\mathbf{E}} + \frac{3}{2}\omega_e r_D^2 \nabla(\nabla \cdot \mathbf{E}) + \frac{\omega_e}{T_e} \left( \frac{1}{8\pi n_0} |\mathbf{E}|^2 - \frac{2(\Delta\alpha)^2}{15T_i} |\mathbf{E}|^4 \right) \mathbf{E} = 0, \quad (9)$$

which has both cubic and quintic nonlinear terms. NLSEs analogous to Eq. (9) were examined previously in connection to the studies of the light bullet propagation in crystals [11].

We shall examine axially or spherically symmetric plasma oscillations, i.e.  $\mathbf{E} = E\mathbf{e}_r$ , where  $\mathbf{e}_r$  is a unit vector in radial direction and  $E$  is a scalar function. Introducing the following dimensionless variables:

$$\begin{aligned} \tau &= \frac{15}{128\pi^2} \frac{T_i}{T_e} \frac{1}{(n_0 \Delta\alpha)^2} \omega_e t, & x &= \frac{1}{8\pi n_0 \Delta\alpha} \sqrt{\frac{5T_i}{T_e}} \frac{r}{r_D}, \\ \psi &= 4\Delta\alpha \sqrt{\frac{\pi n_0}{15T_i}} E, \end{aligned} \quad (10)$$

we can represent Eq. (9) in the form,

$$i\frac{\partial\psi}{\partial\tau} + \psi'' + \frac{d-1}{x}\psi' - \frac{d-1}{x^2}\psi + (|\psi|^2 - |\psi|^4)\psi = 0, \quad (11)$$

which contains no dimensionless parameters. Here  $d = 2, 3$  is the dimension of space.

One can check by the direct calculation that the following quantities:

$$N = \int_0^\infty \Omega_d \, dx \, x^{d-1} |\psi|^2, \quad (12)$$

(the plasmon number) and

$$H = \int_0^\infty \Omega_d \, dx \, x^{d-1} \left( \left| \frac{1}{x^{d-1}} (x^{d-1}\psi)' \right|^2 - \frac{1}{2}|\psi|^4 + \frac{1}{3}|\psi|^6 \right), \quad (13)$$

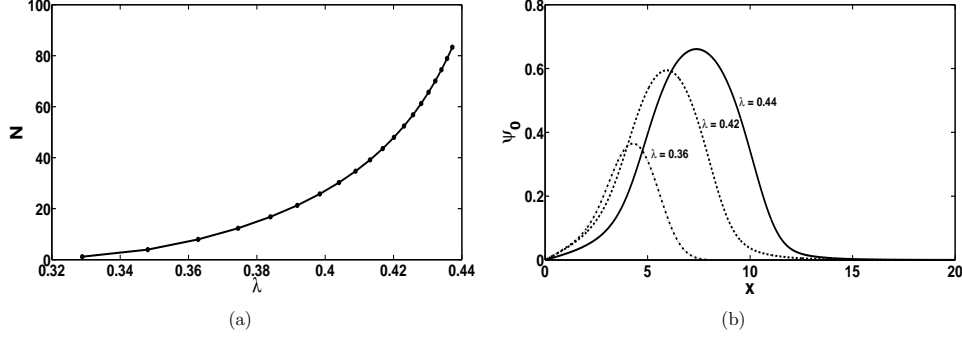


Figure 1: The analysis of Eq. (11) in 2D case. (a) The function  $N(\lambda)$ . (b) Examples of the numerical solitons  $\psi_0(x)$  for different  $\lambda$ . The solid line corresponds to  $\lambda = 0.44$ , the dashed line to  $\lambda = 0.42$ , and the dash-dotted line to  $\lambda = 0.36$ .

(the Hamiltonian) are the integrals of Eq. (11). Here  $\Omega_2 = 2\pi$  and  $\Omega_3 = 4\pi$  are the solid angles in two and three dimensions.

We shall look for the solution of Eq. (11) as  $\psi(x, t) = e^{i\lambda\tau}\psi_0(x)$ , where  $\lambda$  is a real number meaning the dimensionless frequency shift. By the proper choice of the phase we can always make the function  $\psi_0$  to be real. The corresponding ordinary differential equation for the function  $\psi_0$  is solved numerically using the MATLAB program. It requires an appropriate initial guess function  $\psi_g(x)$ . The guess function is taken to have the Gaussian form  $\psi_g = Ax \exp(-x^2/2\sigma^2)$ , with the parameters  $A$ ,  $\sigma$ , and  $\lambda$  chosen to minimize  $H$  (12) at the constant  $N$  (13).

Firstly, we analyze the stability of axially and spherically symmetric solitons by plotting  $N(\lambda)$  dependence. It is shown in Fig. 1(a) for 2D case and in Fig. 2(b) in 3D case. One should notice that in 2D case  $\partial N/\partial\lambda > 0$  in a quite broad range of  $\lambda$ . Thus according to Vakhitov-Kolokolov criterion (VKC) [12] this kind of 2D solitons is stable. In 3D case the accuracy of calculations is significantly lower than that in 2D situation. To build a smooth  $N(\lambda)$  curve in Fig. 2(a) the least squares method was used since the points on this plot, obtained with numerical simulations in MATLAB, have rather big spread, especially at large  $\lambda$ . Meanwhile one can see that unstable and stable solitons coexist in 3D case. In Fig. 2(a) we get that  $\partial N/\partial\lambda < 0$  at  $\lambda \lesssim 0.26$ . Applying VKC we conclude that this branch corresponds to unstable solitons. However, if  $\lambda \gtrsim 0.26$ , we obtain that  $\partial N/\partial\lambda > 0$ , which corresponds to stable solitons.

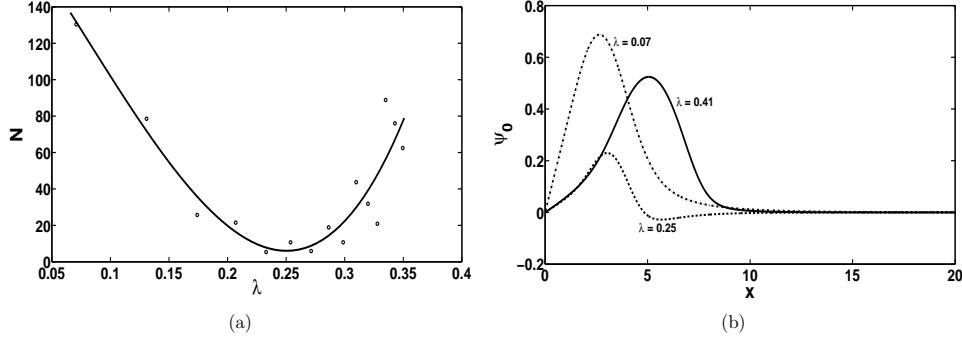


Figure 2: Panels (a) and (b) are the same as in Fig. 1 but correspond to 3D case. In panel (b) the solid line corresponds to  $\lambda = 0.41$ , the dashed line to  $\lambda = 0.25$ , and the dash-dotted line to  $\lambda = 0.07$ .

#### 4. Application

In this section we discuss the application of the results obtained in Secs. 2 and 3 for the theoretical description of a natural atmospheric plasmoid called a *ball lightning* (BL).

BL is a glowing object appearing mainly during a thunderstorm. Despite the existence of numerous BL models (see, e.g. Ref. [13]), it is likely to be plasma based phenomenon. The most spectacular BL property is its long life-time, up to several minutes, which is untypical for an unstructured plasma. In Refs. [14, 15] we put forward a hypothesis that BL is a radial oscillation of plasma and discussed both classical and quantum implementations of this kind of oscillation. Note that analogous idea was considered in Ref. [16]. It is interesting to mention that spherically and axially (snake-like BL, see Ref. [17]) symmetric oscillations of charged particles do not have a magnetic field. Thus this system do not lose energy by electromagnetic radiation.

The main problem of the existence of a plasma oscillation is its stability against the collapse [4]. Various mechanisms for the arrest of the collapse of a Langmuir wave in plasma, such as accounting for electron nonlinearities and additional quantum pressure of electrons, were studied [5, 6, 7].

In the present work we put forward another mechanism for preventing the Langmuir wave collapse based on the interaction of the ions EDM with the oscillating electric field of a plasmoid. We assume that ions in plasma are diatomic and nonpolar. Nevertheless it is allowed for an ion to acquire an induced EDM in the electric field. Thus this situation corresponds to a atmospheric plasma mainly composed of  $N_2^+$  ( $\approx 78\%$ ) and  $O_2^+$  ( $\approx 21\%$ ) ions.

It is known that neither  $N_2^+$  nor  $O_2^+$  possess EDM because of the symmetry reasons. That is why spatial Langmuir solitons studied in the present work are important for the description of stable atmospheric plasmoids.

For the stability of a soliton its size should be greater than  $r_D$  – otherwise thermal electrons will cross the soliton leading to the development of a turbulence. For a rough estimate we shall take the typical ion temperature  $T_i = 300$  K and suppose that the electron temperature in plasma is  $T_e = 10^6$  K. One can see in Figs. 1(b) and 2(b) that in dimensionless variables the typical soliton size can be up to 10. Using the results of Ref. [18] for the polarizabilities of a *neutral* nitrogen molecule  $\alpha_{ij} \sim 10^{-24}$  cm<sup>3</sup> and Eq. (10), we get that for  $n_0 \gtrsim 10^{20}$  cm<sup>-3</sup> a plasmoid becomes stable. This value is close to the density of an ideal gas at standard temperature and pressure, the Loschmidt's number,  $n_l = 2.7 \times 10^{19}$  cm<sup>-3</sup>.

The analysis of the soliton stability is based on the perturbation theory which requires that, cf. Eq. (10),

$$\delta\omega = \lambda \frac{15}{128\pi^2} \frac{T_i}{T_e} \frac{1}{(n_0 \Delta\alpha)^2} \omega_e < \omega_e. \quad (14)$$

Using the above chosen parameters of plasma and  $\lambda \sim 0.4$  in Figs. 1(a) and 2(a), we get that the condition (14) is satisfied if  $n_0 \gtrsim 10^{21}$  cm<sup>-3</sup>, which is one order of magnitude bigger than the previous constraint on  $n_0$ . Nevertheless, one should notice that the nonlinear ponderomotive force  $\mathbf{f}_{\text{pol}}$  in Eq. (4) will be defocusing in any case, providing the soliton stability. If condition (14) is violated, one should just analyze the complete set of nonlinear hydrodynamic equations for plasma rather than use the simplified perturbative treatment.

Finally can estimate the typical size of a plasmoid requiring that  $R_{\text{eff}} > r_D$ . For  $n_0 = 10^{20}$  cm<sup>-3</sup> and  $T_e = 10^6$  K, we get that  $R_{\text{eff}} \gtrsim 10^{-6}$  cm. Such nano-sized plasma structures were discussed in our previous work [14]. Note the obtained tiny core of an atmospheric plasmoid can explain the fact that sometimes BL passes through small holes or cracks in dielectric materials without changing its shape and without destroying surrounding materials.

## 5. Conclusion

In the present work we have studied stable spatial plasma structures possessing axial and spherical symmetry. The stability of soliton-like plasmoids against the collapse is provided by the defocusing interaction of the induced



EDM of ions with the rapidly oscillating electric field. Note that an ion was supposed to be diatomic without static EDM. In Sec. 2, starting from the complete set of plasma hydrodynamic Eqs. (1), (2) and (4) we have derived the basic nonlinear equations for the envelope of the electric field (7) and for the perturbation of the ion density (8). In Sec. 3 we have reduced these equations to a single cubic-quintic NLSE (11), written in the dimensionless variables (10). Then Eq. (11) was solved numerically, cf. Figs. 1(b) and 2(b). Applying VKC, we have found that mainly stable spatial solitons can exist in 2D case, Fig. 1(a), whereas in 3D case both stable and unstable solitons are present, Fig. 2(a).

In Sec. 4 we have discussed a possible application of the obtained results to the theoretical description of a long-lived atmospheric plasma structure, BL. It was found that in a realistic atmospheric plasma, mainly made of  $\text{N}_2^+$  and  $\text{O}_2^+$  ions, one can expect the existence of stable plasmoids described in the present work. Note that  $\text{N}_2^+$  and  $\text{O}_2^+$  are diatomic and can have only induced EDM, i.e. the our approach is valid.

We have found that, for the preventing of a turbulence development, i.e. to guarantee the plasmoid stability, the density of background ions should be greater than  $n_0 = 10^{20} \text{ cm}^{-3}$ , which is very close to the Loschmidt's number. The slightly greater value of  $n_0$  compared to  $n_l$  can be explained by the fact that we have used the polarizabilities of  $\text{N}_2^+$  as for a neutral nitrogen molecule, which in fact can differ significantly. Nowadays no measurements of  $\text{N}_2^+$  polarizabilities have been made.

We have also obtained that the typical plasmoid size is small,  $R_{\text{eff}} \gtrsim 10^{-6} \text{ cm}$ . Such nano-scaled plasma structures were discussed in our previous work [14] where we applied the quantum mechanical approach to explain their stability.

In our estimates of the plasmoid characteristics in Sec. 4, the temperature of the ion component of plasma has been chosen as 300 K, which is an appropriate value. However, to provide the reasonable plasmoid properties, the temperature of the electron gas had to be high,  $T_e = 10^6 \text{ K}$ . We remind that the electron temperature in a linear lightning discharge can be slightly higher than  $\sim 3 \times 10^4 \text{ K}$  [19]. Thus to generate this kind of plasma structures, described in frames of our model, one has to implement rather significant heating of electron gas. We can notice that a plasma with  $n_0 = 10^{20} \text{ cm}^{-3}$  and  $T_e = 10^6 \text{ K}$  may be created by a nano-second laser pulse [20]. This fact is important for the laboratory generation of spherically symmetric plasmoids.

Finally we can also notice that, taking into account the small energy

$\sim 10^2$  J of an object described in frames our model, one may consider this kind of plasmoids as proto-BL, i.e. an atmospheric plasma structure at its initial stages of evolution. Such a plasma oscillation can appear when strong rapidly oscillating electric field, probably associated with a linear lightning, is applied to a natural nano-sized spike, creating a plasma with a very high electron temperature. Under certain conditions, i.e. when other nonlinear effects become important, this proto-BL can then be transformed into a glowing object identified as BL.

### Appendix A. Polarization of a nonpolar diatomic gas in an external electric field

In this Appendix we shall derive the permittivity of a gas of a nonpolar diatomic molecules in an external electric field.

The collective response of plasma results in the concept of “dressed” particles. It means that the Coulomb interaction of a charged particle is replaced by the Debye-Hückel potential,  $\frac{1}{r} \exp\left(-\frac{r}{r_D}\right)$ . If the plasma density is quite high, i.e. the Debye length is short, we may suppose that the electric charge of an ion is quite perfectly screened by surrounding electrons. In this case the Hamiltonian of a diatomic ion possessing an axial symmetry in an external electric field reads

$$H = \frac{\mathbf{M}_\perp^2}{2I} + U, \quad (\text{A.1})$$

where  $\mathbf{M}_\perp$  is the vector of the angular momentum of an ion perpendicular to the ion axis,  $I$  is the moment of inertia of an ion,  $U = -(\mathbf{p}\mathbf{E}) = -\mathbf{E}^2 (\alpha_\perp \sin^2 \theta + \alpha_\parallel \cos^2 \theta)$  is the potential energy of a polarized ion in an external electric field, and  $\theta$  is the angle between the ion axis and the electric field direction.

On the classical level the thermodynamical properties of a gas can be calculated on the basis of the canonical partition function  $Z = z^\mathcal{N}$ , where  $\mathcal{N}$  is the total number of ions. The reduced partition function  $z$  which includes the rotational degrees of freedom of an ion has the form,

$$z_{\text{rot}} = \frac{1}{(2\pi\hbar)^2} \int d^2\mathbf{M}_\perp \exp\left(-\frac{\mathbf{M}_\perp^2}{2IT_i}\right) \times \int d\Omega \exp\left[\frac{\mathbf{E}^2}{T_i} (\alpha_\perp \sin^2 \theta + \alpha_\parallel \cos^2 \theta)\right], \quad (\text{A.2})$$

where  $d\Omega = 2\pi \sin \theta d\theta$  is the solid angle differential.

Calculating the integral over the angular momentum components, we can express  $z_{\text{rot}}$  in the following form:

$$z_{\text{rot}} = \frac{2I}{\hbar^2} T_i \exp\left(\frac{\mathbf{E}^2}{T_i} \alpha_{\perp}\right) z', \quad z' = \int_0^1 dx e^{\xi x^2} = 1 + \frac{\xi}{3} + \frac{\xi^2}{10} + \dots, \quad (\text{A.3})$$

where  $\xi = \mathbf{E}^2 \Delta\alpha / T_i$ . The polarization of the gas in an external electric field can be calculated on the basis of the expression for the free energy  $F = -T_i \ln Z$ , as

$$\mathbf{P} = -\frac{1}{V} \left( \frac{\partial F}{\partial \mathbf{E}} \right)_{T_i} = 2\mathbf{E} n_i \left( \langle \alpha \rangle + \frac{4}{45} \frac{(\Delta\alpha \mathbf{E})^2}{T_i} \right), \quad (\text{A.4})$$

where  $V$  is the volume of a gas. Here we account for the decomposition of  $z'$  in Eq. (A.3).

Basing on Eq. (A.4) we obtain the permittivity of the ion component of plasma

$$\varepsilon = 1 + 8\pi n_i \left( \langle \alpha \rangle + \frac{4}{45} \frac{(\Delta\alpha \mathbf{E})^2}{T_i} \right), \quad (\text{A.5})$$

which was used in Sec. 2 to derive the ponderomotive force acting on ions.

Finally we mention that our calculations are based on the assumption of the constant electric field whereas in Secs. 2 and 3 the electric field is supposed to oscillate with the high frequency  $\sim \omega_e$ . Using the results of Ref. [18] one finds that the typical frequency associated with the polarizability of a molecule is  $\sim 10^{15}$  Hz, which is several orders of magnitude greater than plasma frequencies in realistic plasmas. Thus the approximation of the constant electric field is valid.

## References

- [1] M. Segev, Opt. Quantum Electron. **30**, 503 (1998).
- [2] S. Burger, et al., Phys. Rev. Lett. **83**, 5198 (1999).
- [3] S. V. Antipov, et al., Physica D **3**, 311 (1981).
- [4] V. E. Zakharov, Sov. Phys. JETP **35**, 908 (1972).

- [5] E. A. Kuznetsov, Sov. J. Plasma Phys. **2**, 178 (1976); T. A. Davydova, A. I. Yakimenko, and Yu. A. Zaliznyak, Phys. Lett. A **336**, 46 (2005), physics/0408023.
- [6] M. M. Škorić and D. ter Haar, Physica C **98**, 211 (1980).
- [7] F. Haas and P. K. Shukla, Phys. Rev. E **79**, 066402 (2009), arXiv:0902.3584 [physics.plasm-ph].
- [8] M. Stenhoff, *Ball lightning: an unsolved problem in atmospheric physics* (NY, Kluwer, 1999).
- [9] A. Bergström, Phys. Rev. D **8**, 4394 (1973); I. P. Stakhanov, JETP Lett. **18**, 114 (1973).
- [10] I. E. Tamm, *Fundamentals of the theory of electricity* (Mir Publishers, Moscow, 1979), pp. 162–169.
- [11] A. Desyatnikov, A. Maimistov, and B. Malomed, Phys. Rev. E **61**, 3107 (2000).
- [12] E. A. Kuznetsov, A. M. Rubenchik, and V. E. Zakharov, Phys. Rep. **142**, 105 (1986).
- [13] V. L. Bychkov, A. I. Nikitin, and G. C. Dijkhuis, *Ball lightning investigations*, in *The atmosphere and ionosphere: physics of earth and space environments*, ed. by V. L. Bychkov, G. V. Golubkov and A. I. Nikitin (Springer, Dordrecht, 2010), pp. 201–373.
- [14] M. Dvornikov and S. Dvornikov, in *Advances in plasma physics research*, ed. by F. Gerard, vol. 5 (NY, Nova Science Publishers, 2006) pp. 197–212, physics/0306157; M. Dvornikov, Proc. R. Soc. A **468**, 415 (2012), arXiv:1102.0944 [physics.plasm-ph]; arXiv:1112.0239 [physics.plasm-ph].
- [15] M. Dvornikov, Phys. Scr. **81**, 055502 (2010), arXiv:1002.0764 [physics.plasm-ph]; *ibid.* **83**, 017004 (2011), arXiv:1101.1986 [physics.plasm-ph]; J. Plasma Phys. **77**, 749 (2011), arXiv:1010.0701 [physics.plasm-ph].

- [16] R. Fedele, in *Proceedings of 6th Int. symp. on ball lightning*, ed. by G. C. Dijkhuis, 1999, pp. 126–132; M. L. Shmatov, J. Plasma Phys. **69**, 507 (2003); K. Tennakone, J. Electrostat. **69**, 638 (2011).
- [17] See p. 204 in Ref. [13].
- [18] G. R. Alms, A. K. Burnham, and W. H. Flygare, J. Chem. Phys. **63**, 3321 (1975).
- [19] V. A. Rakov, J. Geophys. Res. **103**, 1879 (1998).
- [20] H. R. Pakhal, R. P. Lucht, and N. M. Laurendeau, Appl. Phys. B **90**, 15 (2008); P. M. Nilson, et al., Phys. Plasmas **18**, 056703 (2011).